A Structural Approach To Information Shares

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Abstract:

We undertake a structural analysis of the Hasbrouck unobserved components and the Madhavan, Richardson, and Roomans microstructure models. We map carefully the relationship between the structural parameters and four alternative measures of price discovery: (1) Hasbrouck; (2) Harris-McInish-Wood; (3) deJong-Schotman; and (4) Yan-Zivot. We describe analytically problems with using each measure: negative information shares; non-uniqueness; and potential violations of market efficiency. Simulation evidence also describes fragile inferences about the uncertainty of share estimates, misleading implications about price discovery, and the pattern of price adjustment. In an application to the Nasdaq dual listing experiment in 2004, we find that price discovery did not shift significantly towards the Nasdaq.

Keywords: microstructure; information share; structural model; MCMC estimation; dual listing;

JEL Codes: G14; G12; D4; C32;

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1. Introduction

Modern security markets are evolving faster than our ability to model them. A key feature has been the development of multiple trading venues for identical or similar securities.

Market microstructure research has examined several aspects of this market fragmentation. Mayhew (2002) finds that multiple listing competition for options lowers bid-ask spreads. Battalio, Hatch, and Jennings (2004) show that fragmentation can result in poor execution quality. Mizrach and Neely (2005) look at interactions between the Treasury futures and spot market.

This paper explores the question of which markets help to disseminate new information about fundamental values, a process known as *price discovery.* The literature has not settled on a single definition of price discovery. Hasbrouck (1995) introduced the concept of *information share.* Harris, McInish and Wood (2002) have proposed an alternative measure. Both concepts rely on alternative decompositions of permanent and transitory innovations and different structural parameters. Lehmann (2002) has considered the merits of each.

Since Lehmann's exploration, there have been two contributions to this literature. deJong and Schotman (2005) introduce a measure similar to a partial R^2. Yan and Zivot (2005) take an alternative approach, arguing that price discovery is a dynamic process. They introduce a fourth measure similar to the impulse response function.

Our contribution is to unify these measures in the context of a structural model for multiple markets. We map carefully from the structural model to the definitions of each type of information share. Researchers working just with the reduced form face potentially misleading inferences. Information shares can be negative. Non-uniqueness from contemporaneous correlation requires ad hoc orderings in the orthogonalization. Unfortunately, the Choleski decomposition implies implausible restrictions on the structural model.

We illustrate these results in two structural models: the Hasbrouck (1995) unobserved components model and the Madhavan, Richardson and Roomans (MRR, 1997) microstructure model. Analytical and simulation evidence reveal these problems to be quite generic, for a wide range of parameter values.

Utilizing Markov Chain Monte Carlo estimation of the structural model, we obtain efficient estimates of uncertainty for the information share measures. We also show that information shares can be high even in markets which move prices *away* from the efficient level.

Our final section introduces an equity market application. We analyze the dual listing initiative on Nasdaq that began in January 2004. We find that Nasdaq's attempt to compete directly with the NYSE did not have much impact on price discovery. We verify this statistically using our MCMC estimates and through comparison to a matched sample.

Section 2 introduces our baseline multi-market unobserved components framework and defines the various information share measures in the context of the structural model. Section 3 discusses conceptual problems with the information shares. Section 4 introduces the MRR model and re-computes information shares. Section 5 discusses estimation and buttresses our analytical results with simulation evidence. Section 6 concludes.

2. The Hasbrouck Unobserved Components (HUC) Model

Much of the development in the literature has taken place in the context of Hasbrouck's (1995) multiple market model. In this framework, there is a single efficient price p_t^* and prices, $p_{i,t}$,in market i fluctuate around it, driven by possibly common shocks. In the econometrics literature, models of this kind are called *unobserved components models.* They have been applied in many other contexts, particularly macroeconomics. In macro, e.g. Morley, Nelson and Zivot (2003), the unobserved component is typically the trend level of output. In the Hasbrouck model, the efficient price is unobserved.

Let p_t be a $N \times 1$ vector of transaction prices for the same asset from different markets and ι is a $N \times 1$ vector of ones,

$$p_t = \iota p_t^* + u_t. \tag{1}$$

The efficient price follows a random walk,

$$p_t^* = p_{t-1}^* + \varepsilon_t, \quad E[\varepsilon_t^2] = \sigma_\varepsilon^2. \tag{2}$$

The error terms to prices, u_t, and the efficient price, ε_t,are allowed to be contemporaneously correlated,

$$u_t = \alpha\varepsilon_t + e_t, \quad E[e_t' e_t] = \Omega, \tag{3}$$

where α is an $N \times 1$ vector, and e_t is a $N \times 1$ vector of error terms.

2.1 Reduced form information shares

The reduced form of the HUC model can be expressed as a first order vector moving average

(VMA) process. The first difference of this model is:

$$\triangle p_t = \epsilon_t - C\epsilon_{t-1}, \tag{4}$$

where ϵ_t is a white noise process with variance-covariance matrix Σ. For a general VMA model, cointegration requires the long run impulse response to be identical for all markets, $C(1) = \iota\theta'$, where θ is a $N \times 1$ vector. For the reduced form model (4), this implies

$$C = (I - \iota\theta'). \tag{5}$$

The Beveridge-Nelson decomposition of (4) enables us to write the price vector as

$$p_t = \iota p_t^* + (I - \iota\theta')\epsilon_t \tag{6}$$

which implies

$$\varepsilon_t = \theta'\epsilon_t. \tag{7}$$

2.1.1 Hasbrouck

Hasbrouck (1995) decomposes the variance of the efficient price (7),

$$Var(\varepsilon_t) = \sigma_\varepsilon^2 = \theta'\Sigma\theta = \sum_{i,j=1}^{N} \theta_i\theta_j\sigma_{ij}, \tag{8}$$

where $\sigma_{ij} = E[\epsilon_{i,t}\epsilon_{j,t}]$.

Assuming that shocks are mutually uncorrelated (i.e. Σ is diagonal), the Hasbrouck information shares are defined as,

$$IS_i^H = \frac{\theta_i^2\sigma_{ii}}{\sigma_\varepsilon^2}. \tag{9}$$

In the case where Σ is non-diagonal, it is common to orthogonalize the system using the Choleski decomposition $\Sigma = C_\Sigma C_\Sigma'$. In such case

$$IS^H = \frac{[C_\Sigma'\theta]_i^2}{\theta'\Sigma\theta}. \tag{10}$$

Unfortunately, C_Σ is not unique for different rankings of the N markets. By varying their order in p_t, we obtain different values of C_Σ and an estimation range for IS_i^H, which we call $IS_{\max}^H$ and $IS_{\min}^H$. In our simulation analysis, we will also report an average of the two, $IS_{avg.}^H$

2.1.2 Harris, McInish and Wood

The approach of Harris, McInish and Wood (2002) relies on an alternative decomposition of market prices. They utilize Gonzalo and Granger's (1995) methodology. This common factor approach orthogonalizes market prices into permanent, g_t, and transitory, h_t, components,

$$p_t = \Phi_1 g_t + \Phi_2 h_t, \tag{11}$$

where Φ_1 and Φ_2 are factor loading matrices. Baillie, Booth, Tse, and Zabotina (2002) show that g_t is a weighted average of current prices $(\iota'\theta)^{-1}\theta$.

Harris, McInish and Wood (2002) propose the market weights on the permanent component as an alternative measure of price discovery,

$$IS_i^{HMW} = \frac{\theta_i}{\iota'\theta}. \tag{12}$$

Note that if the markets are not contemporaneously correlated and their idiosyncratic variances similar, (9) and (12) should be quite similar.

2.2 Information shares from the structural model

There are two alternative measures of information shares which are defined in the context of the structural model rather than the reduced form.

2.2.1 de Jong and Schotman

While both Hasbrouck and Harris, McInish and Wood work with the reduced form equation, de Jong and Schotman (2005) work directly with the structural parameters. They propose a third information share measure that has the interpretation of a partial R^2. The first step is to use the price innovations from the unobserved components model,

$$w_t = \iota\varepsilon_t + u_t = (\iota + \alpha)\varepsilon_t + e_t = \beta\varepsilon_t + e_t, \tag{13}$$

where $\beta = (\iota+\alpha)$. Consider regression of innovation in the efficient price on the shocks to individual prices

$$\varepsilon_t = \gamma' w_t + \eta_t, \tag{14}$$

where η_t innovation in efficient price unrelated to innovations in individual prices, and $var(\eta_t) = \sigma_\eta^2$. We can write the variance in (2) as

$$\sigma_\varepsilon^2 = \gamma'\Upsilon\gamma + \sigma_\eta^2, \tag{15}$$

where $\gamma = \Upsilon^{-1}\beta{\sigma_\varepsilon}^2$ and $\Upsilon = E[w_t w_t'] = {\sigma_\varepsilon}^2\beta\beta' + \Omega$. de Jong and Schotman note that the goodness of fit,

$$R^2 = 1 - \sigma_\eta^2/\sigma_\varepsilon^2 = \textstyle\sum_{i=1}^N \gamma_i\beta_i, \tag{16}$$

can be decomposed by individual markets i. They propose our third information share measure,

$$IS_i^{JS} = \gamma_i \beta_i, \tag{17}$$

as the partial R^2 that is due to each of the N markets.

2.2.2 Yan and Zivot

Yan and Zivot (2005) argue that price discovery is a dynamic process. They propose measuring information shares through the shape of impulse response function of different markets to efficient price innovation, $\frac{\partial p_{i,t+j}}{\partial \varepsilon_t}$. In the long run, due to cointegration $\frac{\partial p_{i,t+j}}{\partial \varepsilon_t} = 1$, which means that after long enough time all markets would incorporate fully innovation to efficient price. But in the short run, different markets can adjust more quickly than others. They emphasize that the Hasbrouck and Gonzalo-Granger measures rely on the effect of transitory deviations of market prices from the efficient price.

To calculate a normalized information share based on the speed of adjustment, Yan and Zivot propose

$$IS_i^{YZ} = \sum_{k=0}^{K^*} L(\frac{\partial p_{i,t+j}}{\partial \varepsilon_t} - 1), \tag{18}$$

where K^* is a truncation lag chosen such that $\frac{\partial p_{i,t+j}}{\partial \varepsilon_t} \approx 1$, and $L(.)$ is a loss function that could be absolute value or mean squared error.

In our simple structural model, the immediate response of market i is $\frac{\partial p_{i,t}}{\partial \varepsilon_t} = 1 + \alpha_i$ and it is equal to 1 for all other periods. In the case when $|\alpha_i|$ is close to zero, innovation to the efficient price is immediately incorporated into the price of the i market. On the other hand, if $|\alpha_i|$ is close to one, it takes just one period for market i to incorporate efficient innovation into the price. The Yan and Zivot reduces to

$$IS_i^{YZ} = \frac{1}{|\alpha_i|} \tag{19}$$

and it does not depend on other structural parameters of the model.

3. Conceptual Problems With Information Shares

We demonstrate the difficulty of working with the reduced form and a subset of structural model parameters.

3.1 Mapping between the reduced form and structural model

The HUC model (3)-(5) can be written in first differences as,

$$\triangle p_t = (\iota + \alpha)\varepsilon_t - \alpha\varepsilon_{t-1} + \triangle e_t, \tag{20}$$

with autocovariance,

$$\Gamma_0 = E[\triangle p_t \triangle p_t'] = \sigma_\varepsilon^2[(\iota+\alpha)(\iota+\alpha)' + \alpha\alpha'] + 2\Omega, \tag{21}$$

$$\Gamma_1 = E[\triangle p_t \triangle p_{t-1}'] = -\sigma_\varepsilon^2\alpha(\iota+\alpha)' - \Omega. \tag{22}$$

The autocovariance structure of the reduced form model (4) is,

$$\Gamma_0 = \Sigma + C\Sigma C', \tag{23}$$

$$\Gamma_1 = -C\Sigma. \tag{24}$$

The mapping between structural model parameters Ω, α, σ_ε^2 and the parameters θ, Σ of the reduced form may done by matching the moments of (21)-(22) and (23)-(24),

$$Var(\varepsilon_t) = \theta'\Sigma\theta = \sigma_\varepsilon^2, \tag{25}$$

$$Cov(\triangle p_t, \varepsilon_t) = \Sigma\theta = \sigma_\varepsilon^2(\iota+\alpha), \tag{26}$$

$$Cov(\triangle p_t, \triangle p_{t-1}) = -(I - \iota\theta')\Sigma = -\sigma_\varepsilon^2\alpha(\iota+\alpha)' - \Omega. \tag{27}$$

These equations imply a unique solution of reduced form parameters in terms of structural parameters,[1]

$$\Sigma = \Omega + \sigma_\varepsilon^2(\iota+\alpha)(\iota+\alpha)', \tag{28}$$

$$\theta = \Sigma^{-1}\sigma_\varepsilon^2(\iota+\alpha). \tag{29}$$

Note that the moments we match do not constrain the sign of the structural parameters. As we will see below, this can imply that either (23) or (29) can be negative.

3.2 Negative information shares

We demonstrate that without structural model restrictions, many of the information shares may be difficult to interpret.

3.2.1 HMW

To simplify the notation we use $\Sigma_i = \Sigma^{-1}$. Assume that there are only two markets, $N = 2$. $IS_i^{HMW} < 0$ iff θ_1 and θ_2 have opposite signs. Without loss of generality let's consider the case

[1] To see it, note that $\sigma_\varepsilon^2\alpha(\iota+\alpha)' + \iota\theta'\Sigma = \sigma_\varepsilon^2\alpha(\iota+\alpha)' + \iota(\iota+\alpha)'\sigma_\varepsilon^2 = \sigma_\varepsilon^2(\iota+\alpha)(\iota+\alpha)'$.

when $\theta_1 < 0$ and $\theta_2 > 0$. The formula for θ_i implies:

$$\sigma_{i,1,1}(1+\alpha_1) + \sigma_{i,1,2}(1+\alpha_2) \quad < \quad 0, \tag{30}$$

$$\sigma_{i,1,2}(1+\alpha_1) + \sigma_{i,2,2}(1+\alpha_2) \quad > \quad 0. \tag{31}$$

Using the definitions of inverse,

$$\sigma_{2,2}(1+\alpha_1) \quad < \quad \sigma_{1,2}(1+\alpha_2), \tag{32}$$

$$\sigma_{1,2}(1+\alpha_1) \quad < \quad \sigma_{1,1}(1+\alpha_2). \tag{33}$$

Then, using the definition of σ_i in terms of structural parameters, implies,

$$(\omega_{2,2} + \sigma_\varepsilon^2(1+\alpha_2)^2)(1+\alpha_1) \quad < \quad (\omega_{1,2} + \sigma_\varepsilon^2(1+\alpha_2)(1+\alpha_1))(1+\alpha_2), \tag{34}$$

$$(\omega_{1,2} + \sigma_\varepsilon^2(1+\alpha_2)(1+\alpha_1))(1+\alpha_1) \quad < \quad (\omega_{1,1} + \sigma_\varepsilon^2(1+\alpha_2)^2)(1+\alpha_2). \tag{35}$$

After simplifying, we obtain

$$\omega_{2,2}(1+\alpha_1) \quad < \quad \omega_{1,2}(1+\alpha_2), \tag{36}$$

$$\omega_{1,2}(1+\alpha_1) \quad < \quad \omega_{1,1}(1+\alpha_2). \tag{37}$$

Consider the following three (non-exhaustive cases) for which information shares will be negative: (1) if $\alpha_1 < -1$, $\alpha_2 > -1$, then $IS_i^{HMW} < 0$ if $\omega_{1,2} > 0$, $\omega_{1,1}$ and $\omega_{2,2}$; (2) if $\alpha_1 > -1$, $\alpha_2 > -1$ or $\alpha_1 < -1$, $\alpha_2 < -1$, then $IS_i^{HMW} < 0$ for $\omega_{1,2} > 0$; (3) if $\alpha_1 > -1$, $\alpha_2 < -1$, then $IS_i^{HMW} < 0$ for $\omega_{1,2} < 0$.

3.2.2 JS

Negative information shares can also arise in the deJong and Schotman measure. For $\theta_1 > 0$ and $\theta_2 < 0$,

$$IS_i^{JS} = \gamma_i \beta_i, \tag{38}$$

where $\gamma = \Upsilon^{-1}\beta{\sigma_\varepsilon}^2$ and $\Upsilon = E[w_t w_t'] = {\sigma_\varepsilon}^2\beta\beta' + \Omega$. Note that $\gamma = \theta$ and $\Upsilon = \Sigma$ in reduced form parameters given definition of β. or in terms of reduced form parameters and normalizing:

$$IS_i^{JS} = \frac{\theta_i(1+\alpha_i)}{\theta_i'(1+\alpha_i)}, \tag{39}$$

Thus for $IS_i^{JS} < 0$ if $\theta_1(1+\alpha_1)$ has opposite sign from $\theta_2(1+\alpha_2)$. Without the loss of generality, we showed requirements for $\theta_1 < 0$ and $\theta_2 > 0$. For IS^{JS}, we have to add that $1+\alpha_i$ should have the same sign, but this is just the second case in the HMW share discussion.

3.2.3 Simulations for Negative IS

The size of the region where IS_i^{HMW} is negative can be very large.

[INSERT Figure 1 HERE]

There is a connection between IS^{HMW} and IS^{JS}. If $(1+\alpha_1)$ and $(1+\alpha_2)$ have the same sign, then $IS_i^{JS} < 0$ and $IS_i^{HMW} < 0$ areas are identical; if they have the opposite sign, then $IS_i^{HMW} < 0$ implies $IS_i^{JS} > 0$ and visa versa. This is result of the fact that to calculate JS, we multiply θ and $1+\alpha$, while for HMW we only use θ.

[INSERT Figure 2 HERE]

3.3 Non-uniqueness in the Hasbrouck IS

Many researchers have already noted that the Hasbrouck IS is unique only if Σ is diagonal. They have not, however, noted the implications of a diagonal Σ for the structural model.

The assumption of diagonal Σ imposes an equality restriction between the structural parameters Ω and α. It holds only if ω_{12} exactly equal to $-\sigma_\varepsilon^2(1+\alpha_1)(1+\alpha_2)$. Note that there is no theoretical justification for relation between covariance of market noise and the product of variance of efficient price innovation and covariances between efficient price innovation and innovation to stock prices on different trading platforms.[2]

Baillie, Booth and Zabotina (2002) have shown that θ is linearly related to the error correction coefficients in the Engle-Granger representation of the reduced form. It follows that $\theta_1 < 0$ implies that the price on market 1 moves in the *opposite* direction on average to the long run equilibrium or efficient price in our case. As a result, the Hasbrouck share, while positive, does not have the desired interpretation if $|\theta_1| > |\theta_2|$ and $\theta_1 < 0$ and $\theta_2 > 0$; it is possible that the Hasbrouck information share will be higher for market 1 even though price is on average moving away from the efficient price. We simulate a case like this in Section 6.

This problem does not arise when Σ is diagonal. For a more general case of non-diagonal Σ, it

[2] Recall that from the mapping between structural and reduced form innovations, $\varepsilon_t = \theta' \epsilon_t$

is possible to have $IS_1 > IS_2$ while $\theta_1 < 0$ and $\theta_2 > 0$ when

$$\left[\sigma_{11}^{1/2}(\sigma_{22}(1+\alpha_1) - \sigma_{12}\sigma_{11}^{-1}(1+\alpha_2))\right]^2 > (\sigma_{11}(1+\alpha_2) - \sigma_{12}(1+\alpha_1))^2(\sigma_{22} - \sigma_{21}^2\sigma_{11}^{-1}), \quad (40)$$

$$\sigma_{22}(1+\alpha_1) < \sigma_{12}(1+\alpha_2), \quad (41)$$

$$\sigma_{12}(1+\alpha_1) < \sigma_{11}(1+\alpha_2). \quad (42)$$

It is easy to see that these conditions hold a wide range of parameters. For example, they hold if $\alpha_2 = -1$, $\alpha_1 < -1$, $\omega_{12} > 0$ and $(\omega_{11} + \sigma_\epsilon(1+\alpha_1)^2)\omega_{22} > \omega_{12}^2$. Figure 3 shows the region where this problem occurs, with the dark area representing cases where the Hasbouck information share is misleading.

[INSERT Figure 3 HERE]

In the next section, we explore to what degree these problems are unique to the HUC model by exploring a more complicated structural model.

4. The MRR Model

We extend the structural microstructure model of Madhavan, Richardson and Roomans (MRR, 1997) to a multi-market setting. The MRR model has additional structure to explain bid-ask spreads and order flow.

4.1 A single market benchmark

Let p_t^* be fundamental value as in the HUC model. To incorporate order flow in the model, define an indicator variable z_t with $z_t = 1$ for a buyer initiated trade, $z_t = -1$ for a seller initiated trade, and $z_t = 0$ for a trade at the quote midpoint. MRR assume that traders have private information that can effect fundamental asset values, which would otherwise follow a random walk,

$$p_t^* = p_{t-1}^* + \theta_z(z_t - E[z_t|z_{t-1}]) + \varepsilon_t, \quad E[\varepsilon_t^2] = \sigma_\varepsilon^2, \quad (43)$$

where $\theta_z \geq 0$ implies that the revision in fundamental value is positively correlated with the innovation in the order flow from market i and measures the permanent impact of the order flow innovation.

Let $\phi \geq 0$ represent the market maker compensation for transaction costs, inventory costs and risk bearing. Prices p_t deviate from the fundamental price p_t^* to compensate for market maker's cost ϕz_t, which increases (decreases) price for buyer (seller) initiated trades,

$$p_t = p_t^* + \phi z_t + u_t, \quad E\left[u_t^2\right] = \sigma_u^2. \tag{44}$$

As with the HUC model, the errors u_t and ε_t are assumed to be mean-zero, normal and are allowed to be contemporaneously correlated,

$$u_t = \alpha\varepsilon_t + e_t, \quad E\left[e_t^2\right] = \sigma_e^2, \tag{45}$$

where α is parameter of correlation. Finally MRR assume that the order flow is autoregressive with errors, ν_t, to close the model,

$$z_t = \rho z_{t-1} + \nu_t. \tag{46}$$

Note that z_t is an observable variable and unrelated to the rest of the system. Given that $Ez_t = 0$ and $Ez_t^2 = (1-\lambda)$, where $\lambda = Pr[z_t = 0]$ it follows that $E\nu_t = 0$ and $\sigma_\nu^2 = E\nu_t^2 = (1-\rho^2)(1-\lambda)$. We will proceed as if $\nu_t \sim N(0, \sigma_\nu^2)$ to simplify derivations. Note that ν_t is white noise with only nine possible values,[3]

$$\nu_t \in [1-\rho, 1+\rho, 1, -1-\rho, -1+\rho, -1, -\rho, \rho, 0].$$

Let us define new variables $\tilde{p}_t = p_t - \phi z_t$ and $\tilde{\varepsilon}_t = \theta\nu_t + \varepsilon_t$, where $E\tilde{\varepsilon}_t^2 = \sigma_{\tilde{\varepsilon}}^2 = \sigma_\varepsilon^2 + \theta_z^2\sigma_\nu^2$. Model (44)-(45) can be written as,

$$\tilde{p}_t = p_t^* + u_t, p_t^* = p_{t-1}^* + \tilde{\varepsilon}_t. \tag{47}$$

We assume that ν_t is independent contemporaneously of any other innovation of the system.

4.2 Multimarket case

In the multimarket case, p_t is a $N \times 1$ vector of asset prices:

$$\begin{aligned}
p_t &= \iota p_t^* + \Phi z_t + u_t, \quad var(u_t) = \Sigma_u && (48)\\
p_t^* &= p_{t-1}^* + \theta_z' \nu_t + \varepsilon_t, \quad var(\varepsilon) = \sigma_\varepsilon^2 && (49)\\
u_t &= \alpha\varepsilon_t + e_t, \quad var(e_t) = \Omega, && (50)\\
z_t &= \Psi z_{t-1} + \nu_t, \quad var(\nu_t) = \Sigma_\nu. && (51)
\end{aligned}$$

where $\Phi = diag(\phi_1, ..., \phi_N)$ is a $N \times N$ diagonal matrix, Ψ is a $N \times N$ matrix of autocorrelation for the flow variable. $z_t = [z_1, ..., z_N]'$ is a $N \times 1$ vector of order flows on different markets, $\theta_z = [\theta_{z,1}, ..., \theta_{z,N}]'$ is a $N \times 1$ vector. In the multivariate case, even though $\nu_{i,t}$ can take only

[3] In estimation, we aggregate transactions over 1/5/10 minutes. Thus $z_t = \frac{1}{t_n}\sum_{i=1}^{t_n} z_{t,i}$ where t_n is the number of transactions during the specified period (1 minute) and $z_{t,i}$ is the direction of order for each transaction. In such a case, assuming Normal distribution of ν_t seems to be an appropriate starting point.

discreet number of values the fact that Ψ is non-diagonal makes the number of such values pretty large, so normal distribution is more appropriate then in the univariate case.

Similar to the univariate case, we define new variables, $\tilde{p}_{i,t} = p_{i,t} - \phi_i z_{i,t}$ and $\tilde{\varepsilon}_t = \theta_z' \nu_t + \varepsilon_t$. Model (48)-(51) can be written as:

$$\tilde{p}_t \quad = \quad \iota p_t^* + u_t, \quad var(u_t) = \Sigma_u \tag{52}$$

$$p_t^* \quad = \quad p_{t-1}^* + \tilde{\varepsilon}_t, \quad \sigma_{\tilde{\varepsilon}}^2 = \sigma_{\varepsilon}^2 + \theta_z' \Sigma_\nu \theta_z \tag{53}$$

Note that the resulting system is similar to the model (1)-(2), with the only exception the process for u_t, $u_t = \alpha(\tilde{\varepsilon}_t - \theta_z' \nu_t) + e_t$.

4.3 Relation between structural model and reduced form

Working with the transformed price series $\tilde{p}_t$ from (52), we can match the moments of the structural and reduced form models. The first differenced series

$$\triangle \tilde{p}_t = (\iota + \alpha)\tilde{\varepsilon}_t - \alpha \tilde{\varepsilon}_{t-1} + \triangle e_t - \alpha \theta_z' \triangle \nu_t, \tag{54}$$

has autocovariances

$$\Gamma_0 \quad = \quad \sigma_{\tilde{\varepsilon}}^2[(\iota + \alpha)(\iota + \alpha)' + \alpha\alpha'] + 2\Omega + \iota\theta_z \Sigma_\nu \theta_z \iota' - (\iota + \alpha)\theta_z' \Sigma_\nu \theta_z \alpha' (\iota + \alpha)' - \alpha \theta_z \Sigma_\nu \theta_z \alpha', \tag{55}$$

$$\Gamma_1 \quad = \quad -\alpha \sigma^2 \tilde{\varepsilon}(\iota + \alpha) - \Omega - \alpha \theta_z' \Sigma_\nu \theta_z \alpha'. \tag{56}$$

Also,

$$Cov(\tilde{\triangle} p_t, \tilde{\varepsilon}_t) = \sigma_{\tilde{\varepsilon}}^2 (\iota + \alpha) - \alpha \theta_z' \Sigma_\nu \theta_z. \tag{57}$$

We then match these three moments to the autcovariances of the reduced form in terms of $\tilde{p}_t$, obtaining

$$Var(\tilde{\varepsilon}_t) = \theta' \Sigma \theta = \sigma_{\tilde{\varepsilon}}^2, \tag{58}$$

$$Cov(\tilde{\triangle} p_t, \tilde{\varepsilon}_t) = \Sigma\theta = \sigma_{\tilde{\varepsilon}}^2 (\iota + \alpha) - \alpha \theta_z' \Sigma_\nu \theta_z, \tag{59}$$

$$Cov(\tilde{\triangle} p_t, \tilde{\triangle} p_{t-1}) = -(I - \iota\theta')\Sigma = -\sigma_{\tilde{\varepsilon}}^2 \alpha (\iota + \alpha)' - \Omega + \alpha \theta_z' \Sigma_\nu \theta_z (\iota + \alpha)'. \tag{60}$$

We again have a unique solution,

$$\theta \quad = \quad \Sigma^{-1}[\sigma_{\tilde{\varepsilon}}^2 (\iota + \alpha) - \alpha \theta_z' \Sigma_\nu \theta_z], \tag{61}$$

$$\Sigma \quad = \quad \Omega + \sigma_{\tilde{\varepsilon}}^2 (\iota + \alpha)(\iota + \alpha)' + \iota \theta_z \Sigma_v \theta_z \iota' - (\iota + \alpha)\alpha \theta_z' \Sigma_\nu \theta_z (\iota + \alpha)'. \tag{62}$$

4.4 Structural model information shares

4.4.1 JS

The first step is to use the price innovations from the unobserved components model,

$$w_t = \iota(\varepsilon_t + \theta_z' \nu_t) + u_t = (\iota + \alpha)\varepsilon_t + \iota\theta_z'\nu_t + e_t = \beta_1\varepsilon_t + \beta_2\nu_t + e_t, \tag{63}$$

where $\beta_1 = (\iota + \alpha)$ is a $N \times 1$ vector, $\beta_2 = \iota\theta_z'$ is a $N \times N$ matrix.

We construct the JS information share by again regressing the innovation in the efficient price on the shocks to individual prices,

$$\tilde{\varepsilon}_t = \gamma' w_t + \eta_t, \tag{64}$$

where the innovation in efficient price η_t is unrelated to innovations in individual prices, and $var(\eta_t) = \sigma_\eta^2$. We can write the variance in (64) as

$$\sigma_{\tilde{\varepsilon}}^2 = \gamma'\Upsilon\gamma + \sigma_\eta^2, \tag{65}$$

where $\gamma = (Ew_t w_t')^{-1} E w_t \tilde{\varepsilon}_t' = \Upsilon^{-1}(\beta_1 {\sigma_\varepsilon}^2 + \beta_2 \Sigma_\nu \theta_z)$ and $\Upsilon = E[w_t w_t'] = {\sigma_\varepsilon}^2 \beta_1 \beta_1' + \beta_2 \Sigma_\nu \beta_2' + \Omega$.

The de Jong and Schotman information share is,

$$IS_i^{JS} = \frac{\gamma_i \zeta_i}{\gamma' \zeta}, \tag{66}$$

where $\zeta = {\sigma_{\tilde{\varepsilon}}^{-}}^2 {\sigma_\varepsilon}^2 \beta_1 + {\sigma_{\tilde{\varepsilon}}^{-}}^2 \beta_2 \Sigma_\nu \theta_z = [\zeta_1, ..., \zeta_N]'$ is a $N \times 1$ vector. Note that if the matrix Σ_ν is diagonal, then

$$\zeta_i = {\sigma_{\tilde{\varepsilon}}^{-}}^2 \sigma_\varepsilon^2 \beta_{1,i} + {\sigma_{\tilde{\varepsilon}}^{-}}^2 \beta_{2,ii} \Sigma_{\nu,ii} \theta_{z,i}, \tag{67}$$

and we can decompose the information share of each individual market $(\gamma_i \zeta_i)$ into a share attributable to the fundamental innovation $\left(\frac{{\sigma_{\tilde{\varepsilon}}^{-}}^2 \sigma_\varepsilon^2 \beta_{1,i}}{\zeta_i}\right)$ and trading frictions $\left(\frac{{\sigma_{\tilde{\varepsilon}}^{-}}^2 \beta_{2,ii} \Sigma_{\nu,ii} \theta_{z,i}}{\zeta_i}\right)$. If Σ_ν is non-diagonal, the decomposition of the IS measure complicated by the covariance terms to each source of information.

4.4.2 YZ

In the MRR structural model, even though each response is immediate, there are two sources of shocks to efficient price. One is the innovation to fundamentals that is unrelated to the order flow; the immediate response of market i is $\frac{\partial p_{i,t}}{\partial \varepsilon_t} = 1 + \alpha_i$ and it is equal to 1 for all other periods. In the case when $|\alpha_i|$ is close to zero, innovation to efficient price is immediately incorporated into price of the i market. On the other hand if α_i close to one, it takes just one period for market i to incorporate efficient innovation into the price.

The second shock is the innovation to order flow. The response of market i to the shock to the order flow on market j is $\frac{\partial p_{i,t+k}}{\partial \nu_{j,t}} = \theta_{z,j} + \phi_i e_i' \Psi^k e_j$, where e_i is a $N \times 1$ vector of zeros with element

i which is equal to 1. Assuming z_t is stationary $\phi_i e_i' \Psi^k e_j = 0$ in the long run and for all markets response is going to be equal to $\theta_{j,z}$.[4]

4.5 Negative information shares

As in the HUC model, it is possible that information shares can have a negative sign in the MRR model. We begin our discussion with the HMW measures.

Let $\mu = \theta_z' \Sigma_\nu \theta_z$. It can be shown that $\theta_1 < 0$ if

$$\omega_{22}\sigma_\varepsilon^2(1+\alpha_1) + \mu(\omega_{22} + \sigma_\varepsilon^2\alpha_2\alpha_2) < \omega_{21}\sigma_\varepsilon^2(1+\alpha_2) + \mu(\omega_{21} + \sigma_\varepsilon^2\alpha_2\alpha_1) \tag{68}$$

and $\theta_2 < 0$ if

$$\omega_{11}\sigma_\varepsilon^2(1+\alpha_2) + \mu(\omega_{11} + \sigma_\varepsilon^2\alpha_1\alpha_1) < \omega_{21}\sigma_\varepsilon^2(1+\alpha_1) + \mu(\omega_{21} + \sigma_\varepsilon^2\alpha_1\alpha_2). \tag{69}$$

To see this, use definition for $\sigma_{\bar{\varepsilon}}^2$ in (53) to obtain

$$\theta = \Sigma^{-1}[\sigma_\varepsilon^2(\iota+\alpha) + \mu]. \tag{70}$$

$$\Sigma = \Omega + \sigma_\varepsilon^2(\iota+\alpha)(\iota+\alpha)' + \mu\iota\iota', \tag{71}$$

where

$$\sigma_{11} = \omega_{11} + \sigma_\varepsilon^2(1+\alpha_1)^2 + \mu, \tag{72}$$

$$\sigma_{12} = \sigma_{21} = \omega_{21} + \sigma_\varepsilon^2(1+\alpha_1)(1+\alpha_2) + \mu, \tag{73}$$

$$\sigma_{22} = \omega_{22} + \sigma_\varepsilon^2(1+\alpha_2)^2 + \mu. \tag{74}$$

The elements of Σ^{-1} are defined as follows

$$\delta_{11} = \frac{1}{d}\sigma_{22}, \tag{75}$$

$$\delta_{12} = \delta_{21} = -\frac{1}{d}\sigma_{12}, \tag{76}$$

$$\delta_{22} = \frac{1}{d}\sigma_{11}, \tag{77}$$

where $d = \sigma_{11}\sigma_{22} - \sigma_{12}\sigma_{21}$ and $d > 0$ for a positive definite matrix.

Then, the sign of θ_2 is determined by the following expression,

$$\theta_2 = \delta_{21}(\sigma_\varepsilon^2(1+\alpha_1) + \mu) + \delta_{22}(\sigma_\varepsilon^2(1+\alpha_2) + \mu), \tag{78}$$

$$\theta_2 d = -\sigma_{21}(\sigma_\varepsilon^2(1+\alpha_1) + \mu) + \sigma_{11}(\sigma_\varepsilon^2(1+\alpha_2) + \mu), \tag{79}$$

$$= -\omega_{21}(\sigma_\varepsilon^2(1+\alpha_1) + \mu) + \omega_{11}(\sigma_\varepsilon^2(1+\alpha_2) + \mu) + \mu\sigma_\varepsilon^2\alpha_1(\alpha_1 - \alpha_2). \tag{80}$$

[4] The process for z_t is stationary unless λ changes over time.

Then, $\theta_2 < 0$ if

$$\omega_{11}(\sigma_\varepsilon^2(1+\alpha_2)+\mu)+\mu\sigma_\varepsilon^2\alpha_1\alpha_1 \quad < \quad \omega_{21}(\sigma_\varepsilon^2(1+\alpha_1)+\mu)+\mu\sigma_\varepsilon^2\alpha_1\alpha_2, \tag{81}$$

$$\omega_{11}\sigma_\varepsilon^2(1+\alpha_2)+\mu(\omega_{11}+\sigma_\varepsilon^2\alpha_1\alpha_1) \quad < \quad \omega_{21}\sigma_\varepsilon^2(1+\alpha_1)+\mu(\omega_{21}+\sigma_\varepsilon^2\alpha_1\alpha_2). \tag{82}$$

We can show a parallel result when $\theta_1 < 0$.

Without loss of generality let $\theta_1 > 0$ and $\theta_2 < 0$, (68)-(69) imply that

$$\omega_{22}[\sigma_\varepsilon^2(1+\alpha_1)+\mu]+\mu\sigma_\varepsilon^2\alpha_2\alpha_2+\omega_{21}\sigma_\varepsilon^2\alpha_1 > \omega_{11}[\sigma_\varepsilon^2(1+\alpha_2)+\mu]+\mu\sigma_\varepsilon^2\alpha_1\alpha_1+\omega_{21}\sigma_\varepsilon^2\alpha_2. \tag{83}$$

Notice that if $\alpha_1 = \alpha_2$, then $\theta_1 > 0$ and $\theta_2 < 0$ whenever $\omega_{22} > \omega_{11}$.

We obtain a similar result for the JS measure. Note that as in HUC, $\gamma = \theta$ and $\Upsilon = \Sigma$. We previously showed that under certain conditions θ_1 and θ_2 can have opposite signs. Thus, if θ_i have opposite signs, for JS_i to have opposite signs, we need that ζ_i have the same signs. Assuming they are positive,

$$\sigma_{\tilde{\varepsilon}}^{-2}\sigma_\varepsilon^2\beta_{1,i}+\sigma_{\tilde{\varepsilon}}^{-2}\theta_{z,1}\sigma_{\nu,11}\theta_{z,2} \quad > \quad 0, \tag{84}$$

$$\sigma_{\tilde{\varepsilon}}^{-2}\sigma_\varepsilon^2\beta_{2,i}+\sigma_{\tilde{\varepsilon}}^{-2}\theta_{z,2}\sigma_{\nu,22}\theta_{z,2} \quad > \quad 0, \tag{85}$$

or

$$\sigma_\varepsilon^2(1+\alpha_1)+\theta_{z,1}\sigma_{\nu,11}\theta_{z,2} \quad > \quad 0, \tag{86}$$

$$\sigma_\varepsilon^2(1+\alpha_2)+\theta_{z,2}\sigma_{\nu,22}\theta_{z,2} \quad > \quad 0. \tag{87}$$

This is always true if $\alpha_1 > -1$ and $\alpha_2 > -1$. Given that $\alpha_1 > -1$ and $\alpha_2 > -1$, $\omega_{22} > \omega_{11}$ is enough to have opposite signs for JS_i.

5. Estimation: The State Space Representation

The estimation of information shares is done in a Bayesian framework using the structural parameter estimates of Ω, α, and σ_ε^2. These parameters are estimated using the state space representation of the models. This allows us to compute all information shares based on a single set of structural parameters. it also ensures that the differences in information shares do not arise from estimation errors. Third, the Markov Chain Monte Carlo (MCMC) algorithm allows us to construct standard errors for parameter estimates.

We can write both structural models in state space form, with an equation for observed prices in each market

$$p_t = Hx_t, \tag{88}$$

and another describing the evolution of the fundamental prices and shocks,

$$x_t = Fx_{t-1} + v_t. \tag{89}$$

We now adapt our two structural models to fit into the state space framework.

5.1 HUC model

We start with the HUC model. For the observation equation, we have

$$H = \begin{pmatrix} \iota & I_{N\times N} \end{pmatrix}, \, x_t = \begin{pmatrix} p_t^* \\ u_t \end{pmatrix}. \tag{90}$$

For the transition equation

$$F = \begin{pmatrix} 1 & 0_{1\times N} \\ 0_{N\times 1} & 0_{N\times N} \end{pmatrix}, \, v_t = \begin{pmatrix} 1 & 0_{1\times N} \\ \alpha & I_{N\times N} \end{pmatrix} \begin{pmatrix} \varepsilon_t \\ e_t \end{pmatrix}, \; E[v_t' v_t] = \begin{pmatrix} \sigma_\varepsilon^2 & \alpha' \sigma_\varepsilon^2 \\ \alpha \sigma_\varepsilon^2 & \sigma_\varepsilon^2 \alpha \alpha' + \Omega \end{pmatrix} \tag{91}$$

Note that in this state space model, the errors of observation and the state equation are not correlated.

In case of HUC we estimate $\hat{\Sigma}_v$. Without loss of generality consider two markets. If we have only two markets this implies 6 structural parameters ($\underset{1\times 2}{\alpha}$, σ_ε^2 and $\underset{1\times 3}{\Omega}$). $\hat{\Sigma}_v$ also contain 6 unique estimates. The structural parameters are uniquely identified using,

$$\hat{\Sigma}_v = \; E[v_t' v_t] = \begin{pmatrix} \sigma_\varepsilon^2 & \alpha' \sigma_\varepsilon^2 \\ \alpha \sigma_\varepsilon^2 & \sigma_\varepsilon^2 \alpha \alpha' + \Omega \end{pmatrix}. \tag{92}$$

This implies

$$\hat{\sigma}_\varepsilon^2 \;=\; \hat{\sigma}_{v,1,1}, \tag{93}$$

$$\hat{\alpha}_i \;=\; \frac{\hat{\sigma}_{v,1,i+1}}{\hat{\sigma}_\varepsilon^2}, \tag{94}$$

$$\hat{\Omega} \;=\; \hat{\Sigma}_{v[2:N+1,2:N+1]} - \hat{\sigma}_\varepsilon^2 \hat{\alpha} \hat{\alpha}', \tag{95}$$

with identification achieved from the fact that all equations have unique solutions.

5.2 MRR model

We need to estimate structural parameters Ω, α, and σ_ε^2, θ, Σ_ν. First note that z_t is observable and ν_t is not related to other innovations, thus $\hat{\Sigma}_\nu$ can be estimated directly from (51).

We can rewrite the MRR model, (48)-(51) in the following state space form:

$$y_t \;=\; Hx_t, \tag{96}$$

$$x_t \;=\; Fx_{t-1} + v_t, \tag{97}$$

where for the observation equation, we have

$$y_t = \begin{pmatrix} p_t - \Phi z_t \\ z_t - \Psi z_{t-1} \end{pmatrix}, \; H = \begin{pmatrix} \iota & I_{N\times N} & 0_{N\times N} \\ 0_{N\times 1} & 0_{N\times N} & I_{N\times N} \end{pmatrix}, \; x_t = \begin{pmatrix} p_t^* \\ u_t \\ \nu_t \end{pmatrix}, \tag{98}$$

and for the transition equation

$$F = \begin{pmatrix} 1 & 0_{1\times N} & 0_{1\times N} \\ 0_{N\times 1} & 0_{N\times N} & 0_{N\times N} \\ 0_{N\times 1} & 0_{N\times N} & 0_{N\times N} \end{pmatrix}, v_t = \begin{pmatrix} 1 & 0_{1\times N} & \theta_z' \\ \alpha & I_{N\times N} & 0_{N\times N} \\ 0_{N\times 1} & 0_{N\times N} & I_{N\times N} \end{pmatrix} \begin{pmatrix} \varepsilon_t \\ e_t \\ \nu_t \end{pmatrix}, \tag{99}$$

$$E[v_t' v_t] = \begin{pmatrix} \sigma_\varepsilon^2 + \theta_z' \Sigma_\nu \theta_z & \alpha' \sigma_\varepsilon^2 & \theta_z' \Sigma_\nu \\ \alpha \sigma_\varepsilon^2 & \sigma_\varepsilon^2 \alpha \alpha' + \Omega & 0_{N\times N} \\ \Sigma_\nu \theta_z & 0_{N\times N} & \Sigma_\nu \end{pmatrix}. \tag{100}$$

Note again that the errors of observation and the state equation are not correlated.

In case of MRR we estimate $\hat{\Sigma}_v$. The structural parameters are uniquely identified using (100) which implies (note that Σ_v in MRR is $2N+1 \times 2N+1$):

$$\hat{\Sigma}_\nu = \hat{\Sigma}_{v[k_1:k_2,k_1:k_2]}, \tag{101}$$

$$\hat{\theta}_z = \hat{\Sigma}_\nu^{-1} \hat{\Sigma}_{v[k_1:k_2,1]}, \tag{102}$$

$$\hat{\sigma}_\varepsilon^2 = \hat{\sigma}_{v,1,1} - \hat{\theta}_z' \hat{\Sigma}_\nu \hat{\theta}_z, \tag{103}$$

$$\hat{\alpha}_i = \frac{\hat{\sigma}_{v,1,i+1}}{\hat{\sigma}_\varepsilon^2}, \tag{104}$$

$$\hat{\Omega} = \hat{\Sigma}_{v[2:N+1,2:N+1]} - \hat{\sigma}_\varepsilon^2 \hat{\alpha} \hat{\alpha}', \tag{105}$$

where $k_1 = N+1$, $k_2 = 2N+1$. Identification again follows from the fact that all equations have unique solutions.

6. Simulations

We present results from three simulations. The first demonstrates the use of MCMC estimation to provide uncertainty bands around the information share estimates. The second illustrates misleading Hasbrouck information shares when the long-run multipliers are negative. The third analyzes the impulse response function for the MRR model.

6.1 Uncertainty of Hasbrouck information shares

We present results for a simulation study of the HUC model in this section. We construct a two market example, $N = 2$. The shocks to the efficient price and individual markets prices are contemporaneously correlated with $\alpha_1 = -0.2$ and $\alpha_2 = -0.5$. The variance of the efficient price is $\sigma_\varepsilon^2 = 0.10$. The covariance matrix of the transition equation is

$$[v_t' v_t] = \begin{pmatrix} 0.0036 & 0 \\ 0 & 0.0026 \end{pmatrix}. \tag{106}$$

Our sample size is $T = 2000$. Results are in Table 1.

[INSERT Table 1 HERE]

Our estimates of the structural parameters are very tight, with the true parameters all lying within the 95% highest posterior density interval (HPDI). Similarly, our MCMC estimates of the information share measures are also within the 95% HPDI of the shares using the true parameters.

Our criticism of the reduced form approach begins with the Hasbrouck measure. The literature commonly refers to the estimated information shares from the alternative Choleski decompositions as the "upper" and "lower" bound for the information shares. This inference is misleading for two reasons. Consider first our estimate of $IS^H_{\min.} = 0.6512$. The MCMC standard error around this estimate is 0.0303. The similar estimates for $IS^H_{\max.}$ are 0.8016 with standard error of 0.0516. Notice that if we move up 2 standard errors from the "minimum" and down 2 from the "maximum", the regions overlap

$$IS^H_{\min.} + 2SE = 0.7247 > 0.6984 = IS^H_{\max.} - 2SE. \tag{107}$$

Notice also that the range of upper and lower bound estimates is only $0.1504 = 0.8016 - 0.6512$ The range including the uncertainty of each upper and lower bound is much larger,

$$IS^H_{\max.} + 2SE - (IS^H_{\min.} + 2SE) = 0.3142. \tag{108}$$

Given that the information shares are often already quite far apart in highly correlated markets, the region of uncertainty is probably much larger than previous researchers have considered.

6.2 Price discovery when markets amplify price deviations

In this simulation, we again set $N = 2$, but market 2 moves the price away from the efficient price on average. This takes place because we assume that $\theta_1 = 0.6579$, error correcting prices in market 1 towards the efficient price, while in market 2, $\theta_2 = -0.6579$ which moves them away. We set all of the idiosyncratic noise terms to have equal variances and assume there is no contemporaneous correlation. By construction, the Hasbrouck information shares in Table 2 are on average 1/2, $IS^H_{avg} = 0.5$.

[INSERT Table 2 HERE]

Market 2, however, is not helping in any way to move prices closer to fundamentals. In fact, by moving prices away from p^*, it amplifies fluctuations in that market. The variance of price in market 2 is higher than the idiosyncratic noise, and nearly three times as high as in market 1.

6.3 Impulse responses in the MRR model

In the MRR structural model, there are two sources of shocks to the efficient price. One is innovation to fundamental price/beliefs that is unrelated to order flow, the immediate response of market i is $\frac{\partial p_{i,t}}{\partial \varepsilon_t} = 1 + \alpha_i$ and it is equal to 1 for all other periods. In the case when α_i is close to zero in absolute value, the innovation to efficient price is immediately incorporated into price of the i market. On the other hand if α_i is close to one, it takes just one period for market i to incorporate the efficient innovation into the price.

The second innovation is to order flow. The response of market i to the shock to the order flow on market j is $\frac{\partial p_{i,t+k}}{\partial \nu_{j,t}} = \theta_{z,j} + \phi_i e_i' \Psi^k e_j$, where e_i is a $N \times 1$ vector of zeros with element i which is equal to 1. Assuming z_t is stationary, $\phi_i e_i' \Psi^k e_j = 0$ in the long run, and for all markets, the response is going to be equal to $\theta_{j,z}$.[5] Figure 4 illustrates shocks from each of these three sources.

[INSERT Figure 4 HERE]

The speed of adjustment to the long run response depends on highest eigenvalue of Ψ; if it is close to 1, the speed of adjustment will take a long time. Also in the case of complex roots the adjustment will fluctuate around long run equilibrium response $\theta_{j,z}$. The predictability of the price path seems hard to reconcile with market efficiency.

7. Application to NYSE-Nasdaq Dual Listing

On January 12, 2004, the Nasdaq began to compete directly with the New York Stock Exchange for listings. Six companies, Apache Corporation (APA), Cadence Design (CDN), Countrywide Financial (CFC), Hewlett-Packard (HPQ), Charles Schwab, Inc. (SCH), and Walgreens (WAG), started to list their shares on both markets. Firms traded under the same three-letter symbol. These six companies had a combined \$156 billion in market cap. This change in listing seems like

[5] The process for z_t is stationary unless λ changes over time.

an ideal natural experiment for testing the effect of listing on information share.

Two competing factors influence the impact of the dual listing. Nasdaq dealers could already trade these stocks through the Intermarket Trading System (ITS). On the other hand, advertising that new markets are available might lead a greater share of price discovery in the new venue.

Our estimates in Table 3 do not reveal any statistically significant change in information share before and after the listing change. It appears that Nasdaq's gain was fairly modest in both market share and information share.

[INSERT Table 3 HERE]

Nonetheless, the results do suggest a small gain for the Nasdaq. Hasbrouck information shares for the NYSE fall for all six stocks, ranging from nearly zero for Schwab and almost 5% for Apache Corp. 17 of the 24 information share estimates fall. The estimates of information shares, in Table 4, are all highly correlated with the exception of Yan-Zivot and Harris-McInish-Wood.

[INSERT Table 4 HERE]

To account for any secular trends in the market at the time, we repeat the exercise with a matched sample of firms, sorting on market cap, trading volume, spreads and price. We wound up with a sample of six companies: IGI Inc. (IG), BMC Software (BMC), Nokia (NOK), SBC Communications (SBC), Southwest Airlines (LUV), and Gillette (G).

The six Hasbrouck information shares in are Table 5.

[INSERT Table 5 HERE]

The NYSE Hasbrouck information share falls only modestly in this period, and the matched sample's decline is not statistically different from the decline in the six dual listed stocks. From the viewpoint of price discovery, the initiative appears not to have been very successful.

Nonetheless, Nasdaq has pushed ahead with its dual listing program. American Financial Group (AFG), Coles Meyer Limited (CM), harmony Gold Mining (HMY), and Ivanhoe Mines (IVN) have been added, along with several mutual funds. Cadence and Schwab have chosen to leave the NYSE. Cadence has been listed exclusively on Nasdaq under the symbol CDNS since October 2005, and Schwab as SCHW since December 2005.

8. Conclusion

We have provided a structural framework for the various estimates of information shares in the literature. The structural model permits restrictions that rule out misleading information share estimates that may arise when working directly with the reduced form.

Estimating the model using Bayesian methods provides correct measures of our uncertainty about the information shares. Simulations show that markets can have high information shares even when they inhibit price discovery.

An application of structural estimation enables us to test statistically that the Nasdaq's dual listing initiative was a success. We do not find a significant change in information share either within the six stocks or as compared to a matched sample.

Our analytical, simulation and empirical evidence, we think, makes a strong case for both structural modeling and Bayesian estimation of the models.

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Table 1
Simulation Results for UC Model - Uncertainty of IS^H

$N = 2$, $T = 2000$	α_1	α_2	σ_ε^2	$\Omega(1,1)$	$\Omega(1,2)$	$\Omega(2,2)$
Structural parameters	-0.2	-0.5	0.1	0.0036	0.0	0.0026
estimates	-0.1863	-0.4782	0.1047	0.0023	-0.0007	0.0027
standard errors	0.0112	0.0106	0.0032	0.0003	0.0002	0.0001
95% HPDI: lower bound	-0.2034	-0.4965	0.0993	0.0018	-0.0010	0.0025
95% HPDI: upper bound	-0.1685	-0.4633	0.1094	0.0027	-0.0003	0.0029

	$IS^H_{\max}$		IS^H_{avg}		$IS^H_{\min}$	
Market $i = 1, 2$	1	2	1	2	1	2
based on true parameters	0.7658	0.2342	0.7163	0.2837	0.6844	0.3156
MCMC mean	0.8016	0.1984	0.7130	0.2870	0.6512	0.3488
MCMC standard error	0.0516	0.0516	0.0058	0.0058	0.0303	0.0303
95% HPDI: MCMC lower bound	0.8016	0.1984	0.7130	0.2870	0.6512	0.3488
95% HPDI: MCMC upper bound	0.8649	0.2654	0.7226	0.2963	0.6979	0.3880

Notes: All statistics are based on MCMC chains of parameters draws: we report mean of MCMC chains, standard errors of MCMC chains and 95% Highest Posterior Density Intervals.

Table 2
Simulation Results for UC Model - Negative θ[6]

$N=2$, $T=2000$	α_1	α_2	σ^2_ε	$\Omega(1,1)$	$\Omega(1,2)$	$\Omega(2,2)$	θ_1	θ_2
Structural parameters	-0.5	-1.5	0.1	0.0026	0.0	0.0026	0.6579	-0.6579

	$IS^H_{\max}$		IS^H_{avg}		$IS^H_{\min}$		$var(\triangle p_{t,i})$	
Market $i=1,2$	1	2	1	2	1	2	1	2
At true parameters	0.3503	0.3503	0.5000	0.5000	0.6497	0.6497	0.0100	0.0277

Notes: We evaluate the information shares at the true parameters. $var(\triangle p_{t,i})$ is the simulated variance of the return series in market i.

[6] All statistics are based on MCMC chains of parameters draws: we report mean of MCMC chains, standard errors of MCMC chains and 95\% Higherst Posterior Density Intervals.

Table 3
Information Shares for Dual Listed Shares

		H	HMW	DS	YZ
APA	2003	0.7190	0.6020	0.7210	0.7410
	2004	0.6704	0.6715	0.7271	0.7277
CDN	2003	0.6705	0.5826	0.7732	0.8079
	2004	0.6460	0.5506	0.6937	0.7682
CFC	2003	0.7474	0.6751	0.7725	0.7973
	2004	0.7312	0.6992	0.7750	0.7752
HPQ	2003	0.6752	0.7066	0.7535	0.7717
	2004	0.6719	0.6879	0.7373	0.7068
SCH	2003	0.6769	0.5726	0.6729	0.7495
	2004	0.6761	0.5997	0.6856	0.7467
WAG	2003	0.6943	0.6750	0.7394	0.7698
	2004	0.6852	0.5903	0.7264	0.7725

Notes: The table lists Hasbrouck (H), Harris, MicInish, and Wood (HMW), deJong and Schotman (DS), and Yan-Zivot (YZ) information shares for the NYSE of the six Nasdaq dual listed stocks. The period in 2003 is prior to dual listing, which began in January 2004.

Table 4
Correlation Among Information Shares for the Dual Listing Sample

	H	**HMW**	**DS**	**YZ**
H	1.0000	0.4239	0.5234	0.2907
HMW		1.0000	0.6271	-0.1303
DS			1.0000	0.5036
YZ				1.0000

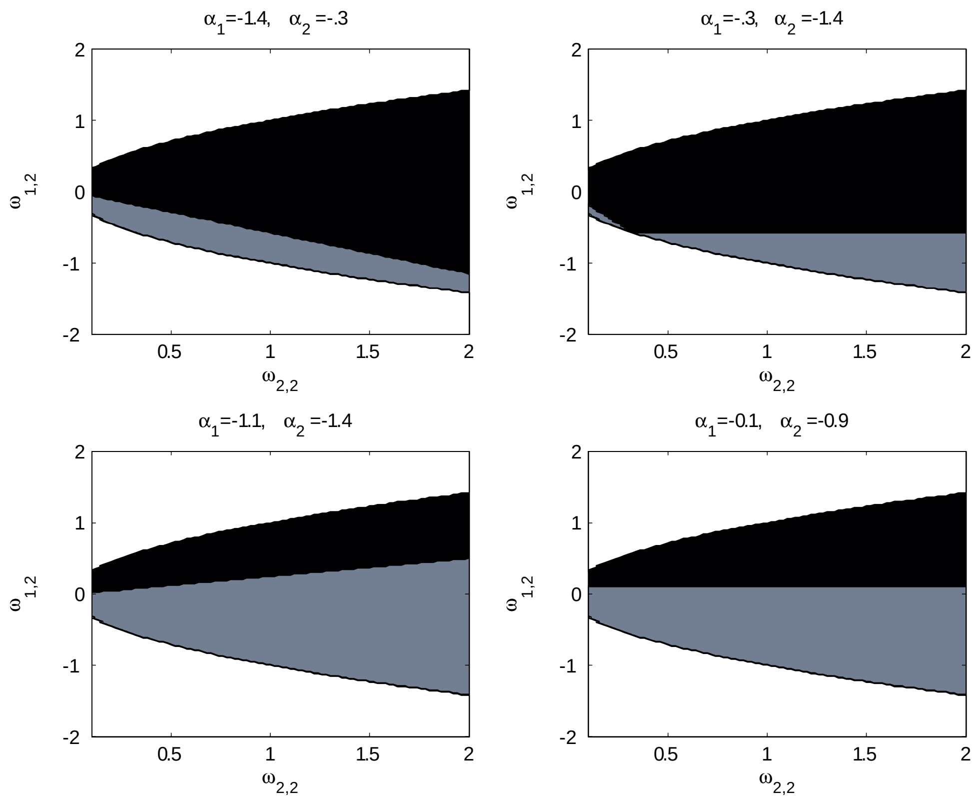
Notes: The table lists the correlation amoung Hasbrouck (H), Harris, MicInish, and Wood (HMW), deJong and Schotman (DS), and Yan-Zivot (YZ) information shares for the six Nasdaq dual listed stocks in 2004.

Table 5
Information Shares for Matched Sample

	NYSE Hasbrouck IS	
Stock	**2003**	**2004**
IG	0.7070	0.6974
BMC	0.7329	0.6994
NOK	0.6696	0.6579
SBC	0.6773	0.6831
LUV	0.7030	0.7128
G	0.7377	0.7280
Avg.	0.7046	0.6964

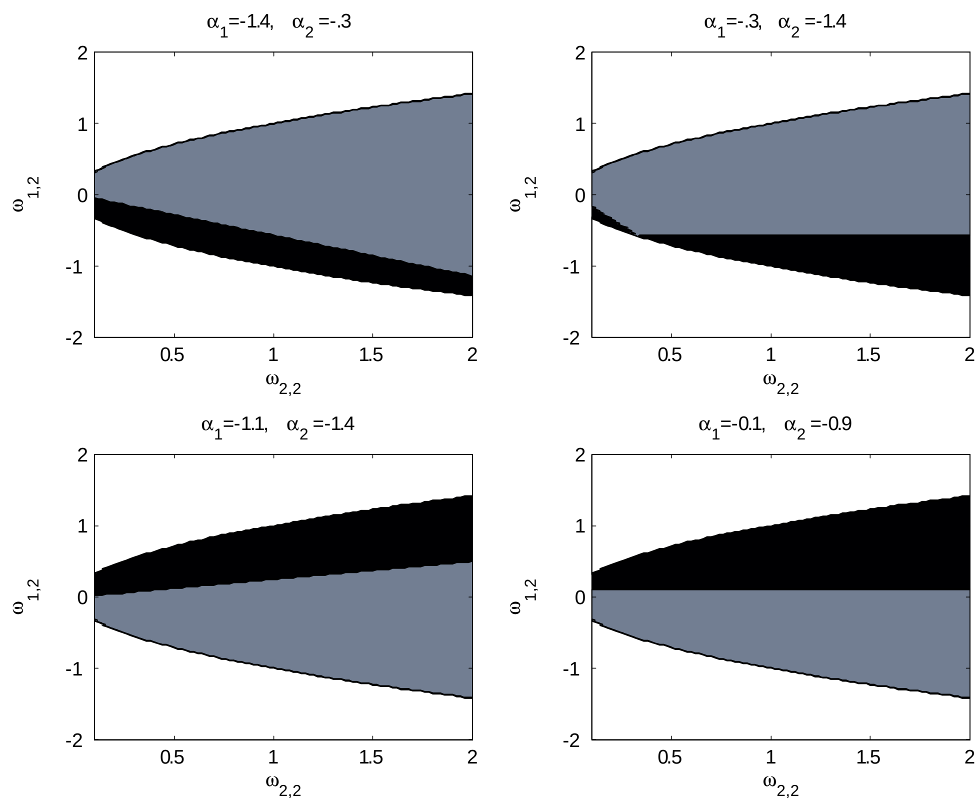
Notes: The table lists Hasbrouck information shares for a matched sample of NYSE firms in 2003 and 2004.

Figure 1
Sign of HMW Information Share



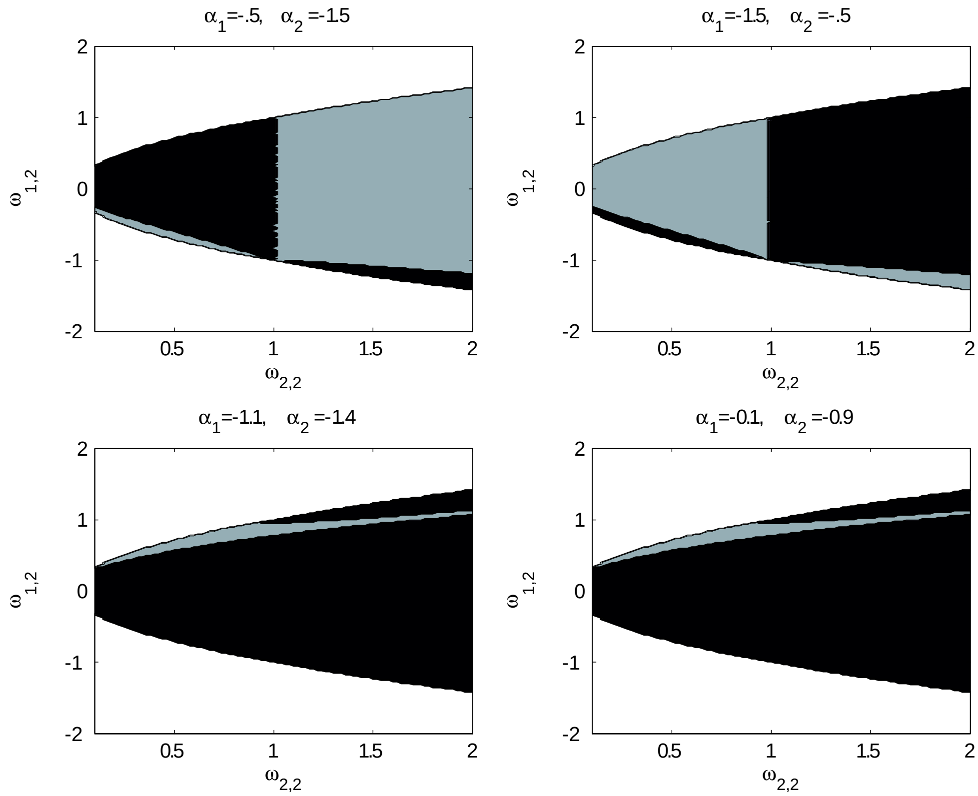
Notes: For all figures $\omega_{1,1} = 1$. For all values of ω_{12}, ω_{22} inside the shaded area, the variance-covariance matrix Ω is positive definite. The dark area represents a negative IS^{HMW}, the gray area denotes positive.

Figure 2
Sign of JS Information Share



Notes: For all figures $\omega_{1,1} = 1$. For all values of ω_{12}, ω_{22} inside the shaded area, the variance-covariance matrix Ω is positive definite. The dark area represents a negative IS^{JS}, the gray area denotes positive.

Figure 3
Hasbrouck Information Share



Notes: The figure illustrates regions where the Hasbrouck share may provide misleading inference. In the dark area, the information share of market 1 is higher than in market 2 even though $\theta_1 < 0$ and $\theta_2 > 0$, implying that the price in market 1 is on average moving away from the efficient price.

Figure 4
Impulse Response Function of the MRR Model